

# Gauge Invariance and Vacuum Energies of Non-Abelian String-Configurations

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We improve on a method to compute the fermion contribution to the vacuum polarization energy of string-like configurations in a non-Abelian gauge theory. We establish the new method by numerically verifying the invariance under (a subset of) local gauge transformations. This also provides further support for the use of spectral methods to compute vacuum polarization energies in general. We confirm that the vacuum energy in the  $\overline{\text{MS}}$  renormalization scheme is tiny as compared to the mass of the fluctuating fermion field. Numerical results for the physical *on-shell* scheme are also presented.

## I. INTRODUCTION

The electro-weak sector of the standard model suggests the existence of extended flux-tubes called  $Z$ - or cosmic strings, which may have profound consequences for cosmological questions [1–4]. These configurations are, however, not protected by any topological argument and thus are classically unstable [5, 6]. To investigate whether quantum effects provide dynamical stabilization, it is very important to compute the vacuum polarization energy  $\Delta E$  of the string configuration *reliably*. In ref. [7] we have recently provided a proof-of-principle computation of  $\Delta E$  in an  $SU(2)_L$  gauge theory in three spatial dimensions<sup>1</sup>. That approach had the drawback that it required the introduction of an auxiliary field at spatial infinity to make various components of the calculation well-defined. In the present letter we demonstrate that the formulation simplifies considerably in a suitable set of gauges. In particular, the use of an auxiliary field at infinity is avoided altogether.

The string configuration is translationally invariant along its symmetry axis (which we choose to be  $\hat{z}$ ), *i.e.* it only depends on the distance  $\rho$  from the axis and the corresponding azimuthal angle  $\varphi$ . Finiteness of the classical energy (per unit length) requires that the string configuration must be pure gauge at spatial infinity, which turns out to have a non-trivial angular dependence due to the winding of the string. As a consequence, gauge variant functionals of the string fields, such as Feynman diagrams, are ill-defined. This is the major obstacle for a straightforward application of the spectral methods [19] to compute the vacuum polarization energy of a string. In ref. [7] this obstacle was circumvented by the introduction of a *return string* that unwound the fields at spatial infinity. In numerical calculations this has the disadvantage that spatial infinity can only be reached by extrapolation to very extended profiles of the return string. These wide extensions induce large impact parameters so that channels with very large angular momenta must be considered. Here we argue that there are particular gauges in which the computation of  $\Delta E$  does not require any return string. The litmus test then is to establish the invariance of  $\Delta E$  under changes of parameters that classify these gauges. We present numerical evidence to confirm that this is indeed the case. Our finding gives further support for the use of spectral methods in general, as it proves the equality of gauge variant and divergent Feynman diagrams, and (equally gauge variant and divergent) terms in the Born series.

In more detail, the string configuration consists of  $SU(2)$  vector and Higgs fields,  $W^\mu$  and  $\Phi$ ,

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<sup>1</sup> See also ref. [7] for a discussion of earlier attempts [8–18] to estimate  $\Delta E$  for string configurations, both in Abelian and non-Abelian theories.

respectively:

$$\begin{aligned}\vec{W} &= n \sin(\xi_1) \frac{f_G(\rho)}{g\rho} \hat{\varphi} \begin{pmatrix} \sin(\xi_1) & i\cos(\xi_1) e^{-in\varphi} \\ -i\cos(\xi_1) e^{in\varphi} & -\sin(\xi_1) \end{pmatrix} \quad \text{and} \\ \Phi &= v f_H(\rho) \begin{pmatrix} \sin(\xi_1) e^{-in\varphi} & -i\cos(\xi_1) \\ -i\cos(\xi_1) & \sin(\xi_1) e^{in\varphi} \end{pmatrix}.\end{aligned}\tag{1}$$

Here, we have used temporal gauge  $W^0 = 0$  and the  $SU(2)$  isospin structure is written in explicit matrix notation. Moreover,  $v$  is the (classical) vacuum expectation value of the Higgs field and  $g$  is the gauge coupling constant introduced in the string parameterization for later convenience.

The configuration (1) is commonly called a Z-string, because the corresponding component  $Z \sim \text{tr}_I(W\tau_3)$  exhibits the spatial dependence of an Abelian string. The radial functions  $f_G(\rho)$  and  $f_H(\rho)$  approach unity at spatial infinity while they vanish for  $\rho = 0$ . They are the typical profiles of the Nielsen–Olesen type of string [20]. The angle  $\xi_1 \in [0, \pi]$  is a free parameter that determines the relative weight of the gauge and Higgs profiles; it also measures the fractional flux carried by the Z-string.

We are mainly interested in the contribution from the fermion fluctuations to the vacuum polarization energy because it dominates the boson contribution when the number  $N$  of other internal degrees of freedom (*e.g.* color) becomes large. Motivated by the standard model we consider a non-Abelian gauge theory in which the gauge field only couples to left-handed fermions. The fermion-string interaction is then given by the Lagrangian

$$\mathcal{L}_\Psi = \bar{\Psi} i\gamma_\mu (\partial^\mu - igW^\mu) P_L \Psi + \bar{\Psi} i\gamma_\mu \partial^\mu P_R \Psi - f \bar{\Psi} (\Phi P_R + \Phi^\dagger P_L) \Psi, \tag{2}$$

where  $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$  are projection operators on right- and left-handed components, respectively. The strength of the Higgs-fermion interaction is parameterized by the Yukawa coupling constant  $f$ , so that the fermions acquire the mass  $m = vf$  via spontaneous symmetry breaking.

## II. CALCULATIONAL TECHNIQUES

We extract the Dirac Hamiltonian from eq. (2) and perform a local gauge transformation  $H \rightarrow U^\dagger H U$  where

$$U = -iP_L \tau_1 \exp(i \hat{n} \cdot \vec{\tau} \xi) + P_R \quad \text{with} \quad \hat{n} = \begin{pmatrix} \cos(n\varphi) \\ -\sin(n\varphi) \\ 0 \end{pmatrix}. \tag{3}$$

Here  $\xi = \xi(\rho)$  is an arbitrary radial function that defines a subset of gauge transformations. The transformed Dirac Hamiltonian becomes

$$H = -i \begin{pmatrix} 0 & \vec{\sigma} \cdot \hat{\rho} \\ \vec{\sigma} \cdot \hat{\rho} & 0 \end{pmatrix} \partial_\rho - \frac{i}{\rho} \begin{pmatrix} 0 & \vec{\sigma} \cdot \hat{\varphi} \\ \vec{\sigma} \cdot \hat{\varphi} & 0 \end{pmatrix} \partial_\varphi + m \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + H_{\text{int}}, \tag{4}$$

$$\begin{aligned}H_{\text{int}} &= m \left[ (f_H \cos(\delta\xi) - 1) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + i f_H \sin(\delta\xi) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \hat{n} \cdot \vec{\tau} \right] + \frac{1}{2} \frac{\partial \xi}{\partial \rho} \begin{pmatrix} -\vec{\sigma} \cdot \hat{\rho} & \vec{\sigma} \cdot \hat{\rho} \\ \vec{\sigma} \cdot \hat{\rho} & -\vec{\sigma} \cdot \hat{\rho} \end{pmatrix} \hat{n} \cdot \vec{\tau} \\ &\quad + \frac{n}{2\rho} \begin{pmatrix} -\vec{\sigma} \cdot \hat{\varphi} & \vec{\sigma} \cdot \hat{\varphi} \\ \vec{\sigma} \cdot \hat{\varphi} & -\vec{\sigma} \cdot \hat{\varphi} \end{pmatrix} \left[ f_G \sin(\delta\xi) I_G(\delta\xi) + (f_G - 1) \sin(\xi) I_G(-\xi) \right].\end{aligned}\tag{5}$$

We have made explicit the dependence on the angles  $\delta\xi \equiv \xi_1 - \xi$  and  $\xi$  via the isospin matrix

$$I_G(x) = \begin{pmatrix} -\sin(x) & -i\cos(x)e^{in\varphi} \\ i\cos(x)e^{-in\varphi} & \sin(x) \end{pmatrix}, \quad (6)$$

while the explicit matrices in eqs. (4) and (5) act in spinor space.

The key idea is now to impose the boundary conditions  $\xi(0) = 0$  and  $\xi(\infty) = \xi_1$ . Together with the boundary conditions for the physical profiles  $f_G$  and  $f_H$  this defines a well-behaved scattering problem for which a scattering matrix and, more generally, a Jost function for momenta in the upper half complex plane can be straightforwardly computed. Furthermore, the Born series to these scattering data can be constructed by iterating  $H_{\text{int}}$ . In contrast to the exact Jost function, the individual terms in this series are gauge dependent, *i.e.* they vary with  $\xi(\rho)$ . After collecting these ingredients we proceed as in ref. [7]:

1. In each angular momentum channel we evaluate the Jost function for imaginary momenta from the Dirac Hamiltonian, eq. (5). To this end we continue the momentum  $k$ , that is conjugate to the radial coordinate  $\rho$ , analytically by substituting  $k \rightarrow i\tau$  with  $\tau$  being a real variable. From the Jost function we then subtract its first and second orders of the corresponding Born series. This difference is summed over angular momenta. The analytic continuation to imaginary momenta is important because it allows us to exchange sums over angular momenta with momentum integrals [14] and implicitly accounts for the bound state contribution to  $\Delta E$ .
2. We introduce a *fake boson* field whose second Born order approximation of the Jost function has the same divergence structure as the combined third and fourth order Born terms for the fermion problem. Again this quantity is summed over angular momenta. The fake boson method is a computational trick to circumvent the very cumbersome evaluation of third and fourth order Born terms and Feynman diagrams. This simplification has been established for purely logarithmic divergent contributions.
3. We integrate the difference of the two functions constructed above over imaginary momenta  $\tau$ , weighted by a kinematical factor characteristic for string-like configurations that are translationally invariant along a fixed direction [21]. The value of this integral is the phase shift contribution,  $\Delta E_\delta$ .
4. We add back the first and second order Born contributions in form of renormalized Feynman diagrams of identical order in  $H_{\text{int}}$ . We call this piece  $\Delta E_{\text{FD}}$  and discuss the details of the necessary counterterms further below.
5. Finally, we add back  $\Delta E_{\text{B}}$  which is the renormalized second order fake boson Feynman diagram that corresponds to the subtraction under 2. It should be emphasized that the renormalization of  $\Delta E_{\text{B}}$  is accomplished by the counterterms in the fermion sector.

In total, the fermion contribution to the renormalized vacuum polarization energy per unit length of the string reads

$$\Delta E = \Delta E_\delta + \Delta E_{\text{FD}} + \Delta E_{\text{B}}. \quad (7)$$

### III. NUMERICAL RESULTS

In addition to the angle  $\xi_1$ , the parameterization of the string background with the above motivated boundary conditions introduces three width parameters,  $w_H$ ,  $w_G$  and  $w_\xi$ ,

$$f_H(\rho) = 1 - e^{-\frac{\rho}{w_H}}, \quad f_G(\rho) = 1 - e^{-\left(\frac{\rho}{w_G}\right)^2} \quad \text{and} \quad \xi(\rho) = \xi_1 \left[ 1 - e^{-\left(\frac{\rho}{w_\xi}\right)^2} \right]. \quad (8)$$

This parameterization guarantees that the interaction Hamiltonian,  $H_{\text{int}}$  is well defined at  $\rho \rightarrow 0$  and no  $1/\rho$  type singularity is encountered. Obviously, the litmus test for our calculation is that the final result for  $\Delta E$  must not depend on the scale  $w_\xi$  introduced in the gauge transformation profile. In our numerical studies we always assume the special case  $n = 1$ .

Numerically the most cumbersome quantity is the phase shift contribution  $\Delta E_\delta$ . For small values of the scale parameters  $w_H$  and  $w_G$ , in particular, the calculation of  $\Delta E_\delta$  for a single background configuration takes several days of CPU time on a modern desktop computer. In the treatment of ref. [7] at least as much time is consumed for *each* set of variational parameters that characterizes the auxiliary return string.<sup>2</sup> Technically, we compute the momentum integral in  $\Delta E_\delta$  with the methods described above only up to a numerical cut-off  $\tau_{\text{max}}$ . For  $\tau > \tau_{\text{max}}$ , we approximate the integrand by an inverse power-law. The numerical cost of this approach is determined by the smallest width parameter in the problem: the smaller this width, the larger we have to take  $\tau_{\text{max}}$  for the power-law approximation to be accurate. Since a larger value for  $\tau_{\text{max}}$  also entails that more angular momentum channels must be summed, the numerics become quickly expensive for small widths. From various integration methods and treatments of the contributions from large angular and linear momenta, we estimate an overall numerical accuracy of 1–2 %, where the upper limit mainly applies to small widths.

#### A. Verification of the method

Before turning to the full string problem we note that the fake boson simplification introduces additional parameters into the numerical calculation. We have numerically verified that these parameters have no effect on the final result.

To verify the method, we will establish the invariance of the vacuum polarization energy within the subset of gauge transformations obtained by varying the width  $w_\xi$  of the gauge transformation profile  $\xi(\rho)$ . It is sufficient to consider the  $\overline{\text{MS}}$  renormalization scheme because any other scheme differs by finite counterterms that are manifestly gauge invariant functionals of the background field. We augment the  $\overline{\text{MS}}$  scheme by the no-tadpole condition which adds the counterterm

$$\mathcal{L}_3 = \frac{c_3}{2} \text{tr}_I \left[ \Phi^\dagger \Phi - v^2 \right] \quad (9)$$

such that the local first order Feynman diagram is exactly canceled. The corresponding result is shown in table I for a typical set of parameters. Since we measure all energies in units of the fermion mass  $m = vf$  and all lengths in its inverse, in the  $\overline{\text{MS}}$  scheme  $\Delta E$  only depends on the specific shape of the background profiles.

We observe that the variation of the total result,  $\Delta E$ , with  $w_\xi$  is significantly less than the estimated numerical error for  $\Delta E_\delta$ , even though some components of  $\Delta E$  change by almost an

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<sup>2</sup> The calculation in ref. [7] needs to be redone several times with varying sets of return string parameters in order to extrapolate to an infinitely distant return string.

$w_\xi$	$\Delta E_{\text{FD}}$	$\Delta E_\delta$	$\Delta E_{\text{B}}$	$\Delta E$
0.5	-0.2515	0.3489	0.0046	0.1020
1.0	-0.0655	0.1606	0.0032	0.0983
2.0	-0.0358	0.1294	0.0038	0.0974
3.0	-0.0320	0.1235	0.0056	0.0971
4.0	-0.0302	0.1193	0.0080	0.0971

TABLE I: Independence on the scale of the gauge transformation parameter. The other parameters are  $w_H = w_G = 2$  and  $\xi_1 = 0.4\pi$ , *i.e.* the gauge field is fairly strong. All energies are given in units of the classical fermion mass  $m = vf$ .

order of magnitude in the considered range<sup>3</sup> of  $w_\xi$ . We find similar results for other variational parameters  $w_H$ ,  $w_G$  and  $\xi_1$ . Within the numerical precision this confirms the gauge invariance of the vacuum polarization energy, at least for the subset of gauge transformation that we have tested.

We have also verified that  $\xi_1 \rightarrow \pi - \xi_1$  leaves  $\Delta E$  unchanged within the numerical precision. This symmetry follows from the fact that for the fields, eqs. (1) this transformation equals a rotation by  $\pi$  about the  $\hat{z}$ -axis in isospace. However, acting with this rotation on the gauge transformation  $U$  in eq. (3) gives a completely different radial function  $\xi(\rho)$  with appropriately modified boundary values. The exact Jost function remains unchanged while neither the Born terms nor the Feynman diagrams are separately invariant, only their combination is.

It should be emphasized again that in the course of this computation, we have added and subtracted formally identical quantities that are *per se* divergent and gauge variant. Hence our study also confirms that their finite pieces are identical, an assertion that is vital for the use of our spectral methods. Previously, such an identity had only be shown for the leading order of the Born and Feynman series within dimensional regularization [19].

The numerical data also confirms our previous result [7] that  $\Delta E$  is very small (as compared *e.g.* against the the fermion mass  $m$ ) within the  $\overline{\text{MS}}$  scheme. Our previous findings were, however, less accurate since they also required an extrapolation to an infinitely distant return string.

In the left panel of figure 1 we display the dependence of  $\Delta E$  (in the  $\overline{\text{MS}}$  scheme) on the angle  $\xi_1$  that characterizes the relative strength of the gauge field and Higgs background. While  $\Delta E(\xi_1)$  is monotonously increasing with  $\xi_1$  (*i.e.* with stronger gauge fields) it develops a minimum around  $\xi_1 = \pi/4$  when the width of the gauge field background is small.

## B. On-shell renormalization

To discuss physical implications we need to impose the on-shell renormalization scheme. For an  $SU(2)_L$  gauge theory the on-shell renormalization conditions and the corresponding determination of the counterterm coefficients have been discussed in ref. [22].<sup>4</sup> To pass from  $\overline{\text{MS}}$  to on-shell, we

<sup>3</sup> Considering even smaller values for  $w_\xi$  becomes numerically even more expensive because the asymptotic behavior of the integrand for  $\Delta E_\delta$  sets in at momenta roughly proportional to  $1/w_\xi$ .

<sup>4</sup> Though the renormalized Feynman diagram is properly displayed in ref. [22], the formula for  $c_4$  is missing an overall factor  $1/2$ .

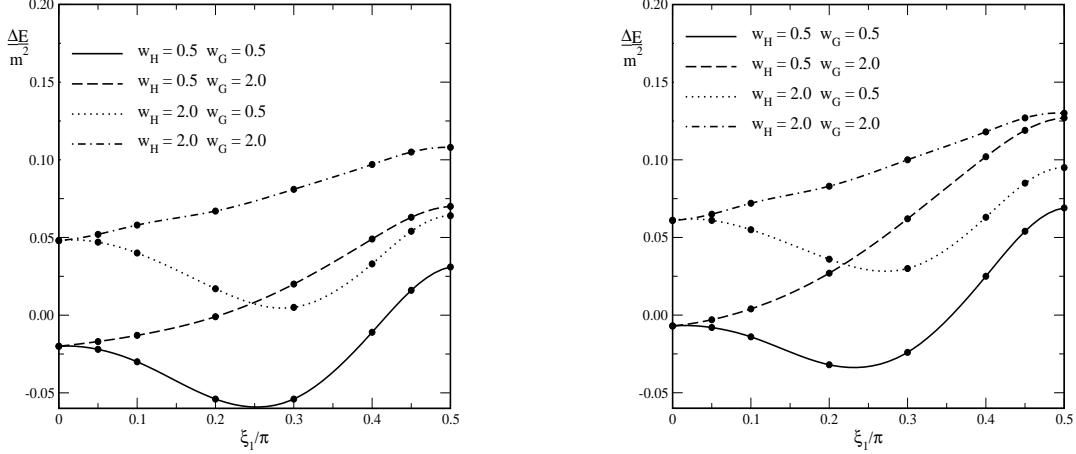


FIG. 1: The parameter dependence of the vacuum polarization energy. Left panel:  $\overline{\text{MS}}$  scheme, right panel: on-shell scheme. The dots denote the computed data and lines are spline interpolations.

have to add the finite and manifestly gauge invariant counterterms

$$\mathcal{L}_{\text{ct}} = c_1 \text{tr}_I [W_{\mu\nu} W^{\mu\nu}] + \frac{c_2}{2} \text{tr}_I \left[ ((\partial_\mu - igW_\mu) \Phi)^\dagger (\partial^\mu - igW^\mu) \Phi \right] + \frac{c_4}{4} \left( \text{tr}_I [\Phi^\dagger \Phi - v^2] \right)^2, \quad (10)$$

where  $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu]$  is the field strength tensor.

The on-shell renormalization condition implies that the pole of the Higgs propagator remains at the tree level mass,  $m_h = m_h^{(0)}$ , with unit residue. This fixes the coefficients  $c_2$  and  $c_4$  and ensures the usual one-particle interpretation of the states created by the asymptotic Higgs field. Furthermore, we also demand that the residue of the gauge field propagator (in unitary gauge) is unity, so that asymptotic  $W$ -fields create one-particle  $W$ -boson states. This condition determines  $c_1$ . The position of the pole in the gauge boson propagator is then a prediction, *i.e.* the physical (on-shell)  $W$ -boson mass receives radiative corrections. In our conventions (with all energies measured in units of  $m = fv$ ) we have  $f = 1/v$ , so that  $f^2 = 2\sqrt{2}m^2 G_F$  makes contact to the standard model parameters. Using  $f = 0.9$  and  $g = 0.7$  approximately reproduces the top-quark and  $W$ -boson masses. Furthermore we use  $\mu_h = m_h/m = v/\sqrt{2} \approx 0.8$ . The corresponding results for  $\Delta E$  are shown in the right panel of figure 1. The additional counterterm contribution in the on-shell scheme increases  $\Delta E$  slightly making its little binding effect from the  $\overline{\text{MS}}$  scheme even smaller.

### C. Total Energy

So far we have not considered the leading contribution to the energy per unit length of the string, *i.e.* the classical energy [13]

$$\frac{E_{\text{cl}}}{m^2} = 2\pi \int_0^\infty \rho d\rho \left\{ n^2 \sin^2 \xi_1 \left[ \frac{2}{g^2} \left( \frac{f'_G}{\rho} \right)^2 + \frac{f_H^2}{f^2 \rho^2} (1 - f_G)^2 \right] + \frac{f_H'^2}{f^2} + \frac{\mu_h^2}{4f^2} (1 - f_H^2)^2 \right\}, \quad (11)$$

where all quantities under the integral are dimensionless. Assuming that there are  $N$  internal degrees of freedom, *e.g.*  $N = 3$  for color, the total energy

$$E = E_{\text{cl}} + N \Delta E \quad (12)$$

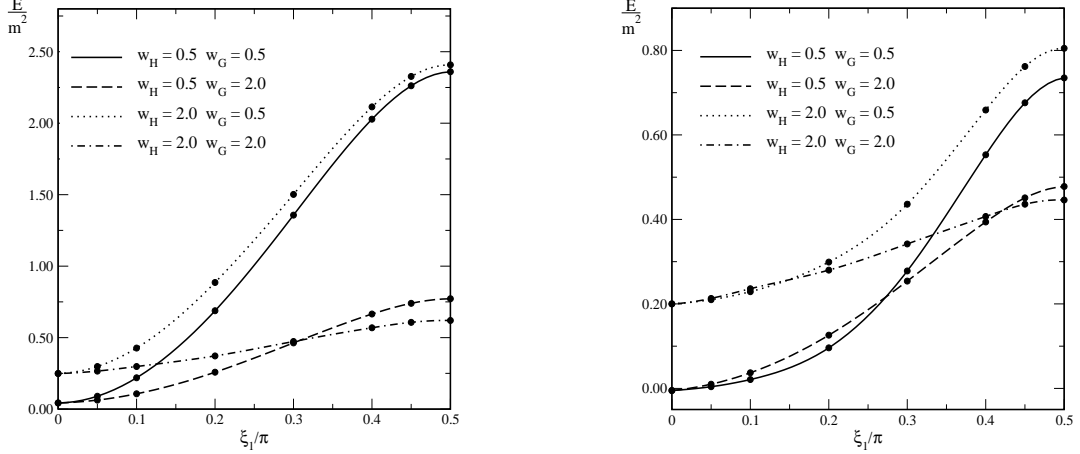


FIG. 2: Total energy, eq. (12) for  $N = 3$ , in the on-shell scheme. Left panel:  $g = f = 5.0$ , right panel:  $g = f = 10.0$ . Note the difference in the scales on the vertical axes.

will always be larger than  $N\Delta E$  at the present level of approximation since  $E_{cl}$  is positive definite. In order to allow for quantum stabilization, the classical energy must be comparable to  $\Delta E$ , *i.e.* tiny as well. This is not the case for standard model motivated parameters, which give  $E_{cl} \sim 10\text{m}^2$ . Eq. (11) shows that small a  $E_{cl}$  requires large coupling constants  $g$  and  $f$ , or equivalently large masses of the fluctuating fermion. To demonstrate this behavior we consider  $E_{cl} + 3\Delta E$  for  $g = f = 5.0$  and  $g = f = 10.0$  in figure 2. For fermion masses of order  $1.5\text{TeV}$  we indeed observe a small binding as  $E < 0$  for narrow fields. However, this may merely reflect the onset of the Landau ghost [23, 24]. It thus seems unlikely that the  $Z$ -string can be stabilized by fluctuating fermions without adding fermion charge.

#### IV. CONCLUSION

We have presented a reliable computation of the vacuum polarization energy that originates from fermion fluctuations about a cosmic string. The present method is significantly more efficient than the only one available so far [7], because it makes redundant the introduction of an auxiliary field near spatial infinity. We have resolved the obstacles that stem from the non-trivial structure of the individual string fields at spatial infinity by choosing a subset of gauges for which the scattering problem is well-behaved. We have verified the novel method by establishing invariance with respect to gauge transformations within this subset. This is far from trivial because in the process of computation formally identical but divergent gauge variant quantities are added and subtracted. As an important side-product we have generated further support for the approach to compute vacuum polarization contributions to observables by spectral methods [19].

Our extensive numerical investigations indicate that the fermion contribution to the vacuum polarization energy produces some binding but it is far too small to overcome the large classical energy and fully stabilize cosmic strings, at least for parameters that are motivated from the standard model.

Another stabilization scenario has been suggested in the  $D = 2 + 1$  model of ref. [13]. Due to symmetry restoration in the core of the string, the Higgs condensate vanishes locally and a significant number of bound states can be induced. Population of these bound states may generate a charged object that is energetically favored against an equal number of free fermions with mass  $m = vf$ . We stress that this binding energy is of the same order in the  $\hbar$ -expansion as the part of



the vacuum polarization energy that we have computed here. Hence the present calculation is a necessary ingredient in a future study of the quantum stabilization of charged cosmic strings. This study will be subject of a future paper that will also serve to provide the details of the present computation.

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